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Then  $3^{a_1+1}-1$  does not contain 5. Hence, the right member contains the factor 5 only in  $11^{a_3+1}-1$ . But this factor must occur at least twice, as  $a_2 \neq 0$ . By writing  $(10+1)^{a_3+1}-1$  and expanding, we may easily show that it contains 5 only once unless  $a_3+1$  is divisible by 5. Then, let  $a_3+1=5n$ . Now,  $11^{5n}-1$  is divisible by  $11^5-1$ , which contains a prime greater than 11. Hence,  $p_3=11$  yields no numbers of the type here considered.

Finally, for  $p_3=13$ , (4) becomes

$$(7) \quad 2^6 \cdot 3^{a_1+1} \cdot 5^{a_2} \cdot 13^{a_3} = (3^{a_1+1}-1)(5^{a_2+1}-1)(13^{a_3+1}-1).$$

If  $a_3+1$  is even,  $13^{a_3+1}-1$  is divisible by  $13^2-1$ . This introduces the inadmissible factor 7. Hence,  $a_3+1$  is odd. The odd powers of 13 end in 3 or 7. Hence,  $13^{a_3+1}-1$  is not now divisible by 5. If  $a_1+1$  is odd,  $3^{a_1+1}-1$  is not divisible by 5. But to satisfy the equation, it must contain 5. Hence,  $a_1+1$  is even, and  $3^{a_1+1}-1$  then contains the factor  $3^2-1=2^3$ .  $5^{a_2+1}-1$  always contains  $2^2$ , and  $13^{a_3+1}-1$  always contains  $2^2$ . Hence, the right member contains  $2^7$ , which is impossible. Therefore, this case yields no numbers of the type here considered.

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## DEPARTMENTS.

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### ALGEBRA.

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250. Proposed by PROFESSOR WILLIAM HOOVER, Ph. D., Athens, Ohio.

Factor  $a^2b^2(x^2+y^2)(a^2y^2+b^2x^2-a^2b^2)=(a^4y^2+b^4x^2)[1/(a^2y^2+b^2x^2)+ab]^2$ .

Solution by the PROPOSER.

Let  $x=r\cos\theta$ ..... (1),  $y=r\sin\theta$ ..... (2); then the given expression equated to zero becomes

$$\begin{aligned} & (b^2\cos^2\theta+a^2\sin^2\theta)[a^2b^2-(b^4\cos^2\theta+a^4\cos^2\theta)]r^2 \\ & \quad -2ab\sqrt{[(b^2\cos^2\theta+a^2\sin^2\theta)](b^4\cos^2\theta+a^4\sin^2\theta)}r \\ & \quad =a^2b^2(b^4\cos^2\theta+a^4\sin^2\theta+a^2b^2) \dots\dots (3). \end{aligned}$$

Multiplying both sides of (3) by the coefficient of  $r^2$  and noticing that

$$\begin{aligned} a^2b^2-(b^4\cos^2\theta+a^4\sin^2\theta) &= a^2b^2(\sin^2\theta+\cos^2\theta)-(b^4\cos^2\theta+a^4\sin^2\theta) \\ &= b^2(a^2-b^2)\cos^2\theta+a^2(a^2-b^2)\sin^2\theta=(a^2-b^2)(b^2\cos^2\theta+a^2\sin^2\theta) \dots\dots (4), \end{aligned}$$

and similarly,

$$a^2b^2+b^4\cos^2\theta+a^4\sin^2\theta=(a^2+b^2)(b^2\cos^2\theta+a^2\sin^2\theta) \dots\dots (5).$$

Completing the square, using the positive sign of the radical,

$$r(a^2 - b^2)(b^2 \cos^2 \theta - a^2 \sin^2 \theta) = ab(a^2 + b^2)\sqrt{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)} \dots\dots\dots (6).$$

Multiplying both sides of (6) by  $r(b^2 \cos^2 \theta - a^2 \sin^2 \theta)$ , squaring, and putting in the values from (1) and (2),

$$(a^2 - b^2)^2 (b^2 x^2 - a^2 y^2)^2 - a^2 b^2 (a^2 + b^2)^2 (b^2 x^2 + a^2 y^2) = 0 \dots\dots\dots (7).$$

This is one factor of the given expression. Using the negative sign after completing the square in (3), and employing (4) and (5),

$$(a^2 - b^2)(b^2 \cos^2 \theta - a^2 \sin^2 \theta)r\sqrt{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)}[r\sqrt{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)} + ab] \\ = 0 \dots\dots\dots (8).$$

Equating the last factor to zero, rationalizing, using (1) and (2), we have  $a^2 y^2 + b^2 x^2 - a^2 b^2$  as a second factor.

Also solved by G. B. M. Zerr.

251. Proposed by S. A. COREY, Hiteman, Iowa.

$$\text{Prove that } \frac{1}{n+1} + \frac{1}{2(n+2)} + \frac{1}{3(n+3)} + \text{etc.}, = \\ \frac{1}{n^2} + \frac{1}{2} \left[ \frac{1}{n-1} + \frac{1}{2(n-2)} + \frac{1}{3(n-3)} + \dots + \frac{1}{l(n-l)} \right],$$

$l$  being equal to  $n-1$ ,  $n$  being any positive integer greater than one.

Solution by L. E. NEWCOMB, Los Gatos, Cal.

$$\text{The general term is, } \frac{1}{r(n+r)} = \frac{1}{nr} - \frac{1}{nr(r+n)}. \quad \text{Let } r=1, 2, 3, \dots\dots\dots \text{ in} \\ \text{succession; then } \frac{1}{n+1} = \frac{1}{n} - \frac{1}{n(n+1)}, \quad \frac{1}{2(n+2)} = \frac{1}{2n} - \frac{1}{n(n+2)}, \quad \frac{1}{3(n+3)} \\ = \frac{1}{3n} - \frac{1}{n(n+3)}.$$

$\therefore$  Sum  $= \frac{1}{n} - \frac{1}{n(n+1)} + \frac{1}{2n} - \frac{1}{n(n+2)} + \frac{1}{3n} - \frac{1}{n(n+3)} \dots\dots\dots$  and all the terms after the  $r$ th vanish.

$$\therefore \text{Sum} = \frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} + \dots + \frac{1}{n^2} \equiv \frac{1}{n^2} + \left[ \frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} + \dots + \frac{1}{ln} \right] (1).$$

In the series (2)  $\frac{1}{n-1} + \frac{1}{2(n-2)} + \frac{1}{3(n-3)} + \dots + \frac{1}{l(n-l)}$ , the general